UNIVERSITY OF ENGINEERING AND TECHNOLOGY

COMPUTER SCIENCE DEPARTMENT

SPATIO-TEMPORAL CRIME PATTERNS VISUALIZATION THROUGH TOPOLOGICAL GRAPH ANALYSIS

RESEARCH WORK

Submitted for partial fulfillment for the Bachelor Degree in Computer Science

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Lima - Peru

21 de octubre de 2025

TABLE OF CONTENTS

INTRODUCTION

Urban crime is a persistent problem that entails considerable challenges for public safety, economic stability and quality of life in modern cities [1]. In Latin American cities like São Paulo, crime is not only frequent but also distributed unequally across space and time [2]. Traditional crime mapping methods, which rely heavily on static heatmaps or aggregated statistics, often fail to capture the dynamic and localized nature of urban criminal activity [3]. This research is motivated by the need for tools that can explore and visualize how crime evolves over geographic space and time [4].

This work investigates how technological approaches, particularly those based on graph theory and data visualization, can reveal hidden structures and patterns in crime data [5]. By modeling the city's road network as a graph and analyzing the distribution of crime over time windows, this research aims to support decision-making and the design of public policies with clearer and more practical insights [6].

This study introduces a framework that combines spatial data modeling with topological analysis and interactive visualization, offering a novel perspective to understand and monitor urban crime dynamics.

Problem statement

Despite the growing availability of geolocated crime data, most urban crime analysis tools still rely on aggregated statistics or static visualizations that fail to reflect the complex spatial and temporal dynamics of criminal activity (Levine, 2006 [7]). These approaches overlook localized trends, delay the detection of emerging hotspots and provide limited support for real-time decision-making (Ratcliffe, 2010 [8]).

In a large and complex city like São Paulo, crime is not randomly distributed, it clusters in specific areas and varies significantly over time. Without tools that enable the detection and visualization of these shifting patterns across space and time, policymakers and public safety institutions are left with an incomplete understanding of crime dynamics. Additionally, current models rarely account for the topological structure of urban road networks, which plays a critical role in how crime spreads and persists in certain areas (Porta, 2006 [9]).

There is a clear need for computational methods that integrate spatial topology, temporal segmentation and interactive visualization to better capture, interpret and act upon urban crime data. This research addresses that gap by proposing a topological framework for the spatio-temporal analysis of crime in São Paulo.

Research objectives

General Objective To develop a topological and interactive framework for the spatio-temporal analysis of urban crime patterns in São Paulo, enabling the identification and visualization of the evolution of crime clusters over time and by type of offense.

Specific Objectives

- Model the urban environment of São Paulo as a street network (graph G) and define monthly scalar functions based on crime density, segmented by crime type.

- Compute metrics such as the volume and hypervolume of the subgraphs to quantify the size and persistence of crime hotspots.
- Compute topological structures (split trees) from the monthly scalar fields to extract a set of significant, localized spatio-temporal subgraphs ($k = 10$ per month).
- Cluster the resulting 1,440 subgraphs using dimensionality reduction (UMAP) and density-based (DBSCAN) algorithms to identify structurally and spatially similar crime patterns.
- Develop and apply a novel ranking metric ($M_c = \mu/\sigma$) to quantify and rank the identified clusters based on their mean intensity and temporal stability.
- Develop an interactive visualization tool that links the temporal timeline, geographic map, and cluster ranking to enable dynamic exploration of persistent and volatile crime patterns.

Justification

This research addresses that gap by integrating tools from Topological Data Analysis (TDA), unsupervised machine learning and interactive visualization. **Its innovation lies not only in using TDA** to extract structural hotspots (subgraphs) from the urban network, **but also in clustering these subgraphs over a 12-year period** to discover persistent, recurring patterns that traditional methods would miss. In doing so, this project offers not only a methodological contribution but also a practical tool that can be adapted to other urban contexts, particularly in Latin American cities facing similar security challenges.

Contribution

- A novel pipeline is proposed that combines Topological Data Analysis (TDA) for feature extraction and machine learning (UMAP + DBSCAN) for pattern clustering.
- A web platform is developed to dynamically explore the evolution of crime patterns, linking cluster rankings with their temporal and spatial distributions.
- A novel ranking metric (M_c) is proposed to quantify the temporal persistence and stability of crime clusters, allowing for the identification of "Top 5"(chronic) and "Bottom 5"(volatile) patterns.
- A flexible framework is presented, which can be adapted to other cities or integrated with real-time data to support prevention strategies and public policy decision-making.

CHAPTER I

THEORETICAL FRAMEWORK

1.1 Spatio-temporal crime analysis

- **Hotspot:** A crime hotspot is defined as a delimited area with a statistically significant concentration of criminal incidents during a specific time period. In various studies, crime hotspots are identified at micro-level units such as street segments or intersections (e.g., Herrmann, 2015 [10]; Sherman, 1989 [11]), while others use grid cells or neighborhood-scale units (e.g., Hu, 2018 [12]; Law, 2015 [13]). The definition inevitably incorporates a temporal dimension, whether by time of day, seasonal patterns, or annual trends, so that the hotspot is not simply a fixed location, but rather an area whose intensity changes over time. For example, Herrmann (2015) [10] and Tompson and Townsley (2010) show that the intensity and compactness of hotspots may vary by time of day, and Hu (2018) [12] demonstrates short-term clustering within a 14-day window.

- **Temporal windows:** The use of temporal windows enables the analysis of crime behavior across different time scales, such as days, weeks, or months. This technique involves segmenting the dataset into specific intervals to observe changes, cyclical patterns, or anomalous events in criminal activity. Broader windows (e.g., monthly) help reveal overall trends, while narrower ones (e.g., daily) can uncover operational details useful for immediate intervention. In spatio-temporal studies, such divisions are key for constructing sequences that allow comparison of hotspot evolution and the dynamics of crime expansion or contraction (Andresen and Malleson, 2013 [3]; Ratcliffe, 2010 [8]).

1.2 City representation and modeling

- **Street network as a graph:** Representing a city as a graph is a foundational technique for computational urban space analysis. In this model, intersections are represented as nodes and streets as edges, allowing the structural connectivity of the road network to be captured. This approach has been widely adopted in studies of mobility, accessibility, and urban crime, as it facilitates the integration of spatial metrics such as centrality or geodesic distance between points (Porta, 2006 [9]; Barthelemy, 2011 [14]).

- **Scalar function:** A scalar function is a mathematical assignment that associates a numerical value to each node (in some cases, each edge) in a graph, representing a quantifiable property of the modeled space (Tierny, 2017 [6]). In urban networks, scalar functions enable the overlay of contextual information, such as crime intensity, traffic flow, or population density, onto the topological structure of the city. This combination of structure and value allows for a shift from purely geometric to functional representations of urban space (Edelsbrunner and Harer, 2010 [15]). The use of scalar functions in urban graph analysis has proven to be a powerful tool for conducting multiscale analyses and producing meaningful visual summaries of the dynamic behavior of complex social phenomena such as crime (Gao, 2022 [16]; Hilaga, 2001 [17]).

1.3 Topological analysis

- **Join and split trees:** Join trees and split trees are topological structures used to describe how the connected components of a scalar function change as the scalar threshold varies. In an urban context such as crime analysis, these trees allow us to represent how regions of high or low crime intensity join or split as we traverse the scalar values. This approach makes it possible to identify hierarchical and dynamic patterns of crime concentration (Carr, 2003 [18]; Tierny, 2012 [19]).

- **Volume and hypervolume:** The volume of a subgraph refers to the geographic area covered by its nodes (e.g., total area), while the hypervolume also incorporates the temporal or scalar dimension, measuring how long a crime pattern "persists" over time or across different intensity levels (Wang, 2020 [20]). This metric is key to quantifying the structural significance of certain crime patterns and comparing their stability across different areas (Chazal et al., 2014 [21]; Bubenik, 2015 [22]).

- **Topological persistence:** Topological persistence is a concept from computational topology that measures how long a topological feature persists across a given scale (e.g., thresholds of a scalar function) (Ge, 2021 [23]). In this context, it is used to identify relevant and stable structures—such as crime clusters—that do not disappear with slight variations in the threshold. Persistence diagrams help distinguish between noise and robust patterns (Edelsbrunner and Harer, 2010 [15]; Cohen-Steiner, 2007 [24]).

CHAPTER II

STATE OF THE ART

The analysis of spatio-temporal crime patterns has evolved over the past decades at the intersection of data science, criminology and urban studies. Traditional techniques such as heatmaps or kernel density estimation (KDE) often rely on aggregated spatial units (e.g., neighborhoods, districts) which, while effective at a macro scale, fail to accurately capture crime dynamics at the street or intersection level. Moreover, they tend to focus on intensity rather than persistence or temporal evolution of the phenomenon (Chainey and Ratcliffe, 2008 [8]; Levine, 2006 [9]).

2.1 Topological approaches

Several recent studies have incorporated topological structures and network-based modeling to more richly represent crime concentration. Lukasczyk (2015) [5] introduced Reeb graphs as a tool to visualize the evolution and duration of crime hotspots, revealing their periodicity and spatial hierarchy. Wang (2017) [25] proposed an approach that combines optimized spatio-temporal KDE with directed acyclic graphs to track the movement and duration of criminal clusters. Hagenauer (2011) [26] integrated spatio-temporal scan statistics with self-organizing maps (SOMs) to represent trajectories and complex hotspot contexts. Phillips and Lee (2009, 2012) [27] employed spatial correlation and graph-based methods to identify co-occurring crime patterns.

These methods enable the identification of not only critical zones but also how these zones connect, evolve, and persist over time.

2.2 Community detection

Modeling crime as a network allows for the representation of complex relationships between locations, events and time. Moreno (2021) [1] employed dynamic network analysis with centrality metrics and community detection techniques to identify key nodes and persistent clusters in Bogotá's criminal activity. Davies and Marchione (2015) [28] applied motif analysis to detect three-node patterns, showing that crimes tend to cluster within constrained spatio-temporal domains. Qazi and Wong (2017) [29], in turn, developed knowledge graphs with tree structures and graphical icons to represent criminal hierarchies and support hypothesis generation in policing contexts.

2.3 Probabilistic detection

The CriPAV (Crime Pattern Analysis and Visualization) model represents an advanced hybrid approach that integrates probabilistic modeling, deep learning and urban networks (Garcia, 2021 [30]). By using a stochastic matrix derived from co-occurrence time series and an embedding model based on autoencoders (Hotspot2Vec), CriPAV groups hotspots according to their temporal behavior. This tool improves interpretability and visualization of street-level crime patterns, identifying not only crime intensity but also the likelihood of recurrence.

2.4 Topological analysis in mobility data

Doraiswamy et al. (2014) [31] introduced TopoMap, a technique based on join and split trees applied to taxi trajectory data to detect spatio-temporal patterns. Their method enables the identification of high-activity zones, convergence points and topological bifurcations. Although originally designed for mobility analysis, this approach is directly applicable to geolocated crime events, especially in contexts where density and persistence are key to interpretation.

CHAPTER III

REFERENTIAL FRAMEWORK

3.1 Legal Framework

The crime data underpinning this research is sourced from the **Núcleo de Estudo da Violência (Center for the Study of Violence) at the University of São Paulo (NEV-USP)** [32]. The NEV is a leading academic research center that curates, standardizes and anonymizes official crime records obtained from governmental bodies, such as the São Paulo Secretariat of Public Security (SSP).

The use of this dataset is governed by Brazil's legal framework for data protection and access to information. While the *Lei de Acesso à Informação* (LAI, Law No. 12,527/2011) mandates public agencies to provide data, the *Lei Geral de Proteção de Dados* (LGPD, Law No. 13,709/2018) imposes strict rules on the processing of personal information.

Our methodology adheres to these regulations by using the NEV-USP dataset, which is specifically prepared and disseminated for academic and research purposes. The data is aggregated and anonymized, ensuring that no personally identifiable information is processed, in full compliance with the ethical and legal standards for research stipulated by the LGPD. This curated academic dataset provides a robust and reliable foundation for our spatio-temporal analysis.

3.2 Geographical Framework

The city of São Paulo presents a complex and highly unequal urban environment, making it a critical case study for spatio-temporal crime analysis. As a megacity, it is characterized by significant socio-economic disparities and pronounced spatial segregation. These inequalities, which often delineate central affluent districts from vast peripheral zones (*periferias*), are strongly correlated with the distribution and nature of crime. Studies on the geography of homicide and other offenses in São Paulo have confirmed that crime patterns are not random. Instead, they are influenced by local socio-economic conditions and urban features, such as the concentration of favelas, commercial zones or major transport nodes, which create criminogenic conditions.

3.3 Historical Framework

The period of analysis for this study, 2006 to 2017, falls within a notable era for public security in São Paulo. While the state had already experienced a significant and widely studied decline in homicide rates from the highs of the 1990s [33], other crimes, such as robbery and theft, remained persistent public challenges [34]. This shift in crime dynamics, coupled with the established presence of organized crime groups like the Primeiro Comando da Capital (PCC), has fundamentally shaped the public security landscape [33, 35]. The state's public security policies during this time continued to evolve, focusing on data-driven policing and targeted interventions. Understanding these historical trends is crucial for interpreting the temporal evolution and stability of the crime clusters identified in our analysis.

CHAPTER IV

METHODOLOGICAL FRAMEWORK

This chapter details the computational pipeline designed to identify, extract and analyze spatio-temporal crime patterns from incident-level data. The methodology transforms raw crime data into a series of monthly topological structures, from which significant çrime event”subgraphs are extracted. These subgraphs are then clustered to reveal recurring patterns, which are subsequently ranked and visualized.

4.1 Urban Graph construction

The foundational spatial structure for this analysis is the street network of São Paulo. We first defined a 7,500-meter radius around a central point in the city (-23.588165, -46.602840). Using the **OSMnx** library [36], we downloaded the ’walk’ (pedestrian) street network graph within this radius from OpenStreetMap.

This raw graph was then simplified using `osmnx.simplify_graph` to remove non-intersection nodes, creating a topologically corrected graph, G. This graph $G = (V, E)$ serves as the static base map, where V represents street intersections and E represents street segments, upon which all crime data is analyzed.

4.2 Topological Data Processing and Event identification

Instead of analyzing raw crime points, we first process the monthly crime data using Topological Data Analysis (TDA). This process models the crime density as a scalar field over the graph G. This scalar field is analyzed to produce topological structures known as *join trees* and *split trees* for each month in the dataset (January 2006 to December 2017).

- A **join tree** captures the merging of connected components, typically corresponding to the peaks or hotspots of crime density.
- A **split tree** captures the splitting of components, corresponding to the valleys or local minima of crime density.

These trees (`join_tree_YYYY-MM.pkl` and `split_tree_YYYY-MM.pkl`) represent the hierarchical structure of crime hotspots and coldspots for each month.

4.3 Spatio-Temporal Subgraph generation

To create a dataset of comparable crime patterns, we systematically extract significant local event structures from the monthly topological trees. This process is executed for each of the 144 months in the study period:

1. **Event Significance (Hypervolume):** For each month's `split_tree`, we compute a *hypervolume* for each node (event) using the `compute_hypervolume` function. This metric sums the scalar values of a node and its descendants, quantifying the importance or persistence of that local minimum (valley) in the crime neighbors.
2. **Top-k Event Selection:** We select the $k = 10$ most significant "minima.^{ev}ents from each monthly `split_tree` based on their hypervolume (`select_top_k_events`). These k nodes represent the most prominent crime "valleys"for that month.

3. **Subgraph Extraction:** For each of these k event nodes, we extract its corresponding local neighborhood from the main urban graph G. This creates a small, localized subgraph representing the spatial structure of that specific crime event.

This process results in a large dataset of 144 months $\times$ 10 events/month = 1 440 subgraphs. Each subgraph is stored with its associated nodes, edges, spatial centroid, and parent month.

4.4 Subgraph vectorization and clustering

To identify recurring patterns, we cluster this dataset of 1,440 subgraphs.

1. **Feature Engineering (Vectorization):** Each subgraph is transformed into a fixed-length feature vector. Based on the provided code, these features are:
   - `num_nodes`: The number of nodes in the subgraph.
   - `num_edges`: The number of edges in the subgraph.
   - `centroid_lat`: The latitude of the subgraph's geometric centroid.
   - `centroid_lon`: The longitude of the subgraph's geometric centroid.
2. **Dimensionality Reduction (UMAP):** This 4-dimensional feature space is then reduced to a 2-dimensional embedding using **Uniform Manifold Approximation and Projection (UMAP)** (`umap.UMAP`) [37]. UMAP is a non-linear technique adept at preserving the global structure of the data in a low-dimensional space, making it suitable for subsequent clustering.
3. **Clustering (DBSCAN):** We apply the **DBSCAN** (`sklearn.cluster.DBSCAN`) algorithm [38] to the resulting 2D UMAP coordinates. DBSCAN is a density-based algorithm chosen for its ability to find arbitrarily shaped clusters and

identify noise (subgraphs that do not belong to any coherent pattern). This step assigns a cluster ID to every subgraph, grouping structurally and spatially similar crime events.

4.5 Cluster Ranking Metric

After clustering, we apply the descriptive metric formulated to rank the resulting clusters. This metric identifies which clusters represent the most intense and stable (or unstable) crime patterns.

For a given cluster c, which contains a set of subgraphs, the metric M_c is calculated by first finding the mean (μ) and standard deviation (σ) of crime counts for *each subgraph* in the cluster across its *temporal window*. The final metric is the average of this ratio for all subgraphs in the cluster:

$$M_c = \frac{1}{|c|} \sum_{G_t \in c} \frac{\mu(G_t)}{\sigma(G_t)} \tag{IV.1}$$

A high M_c value suggests a cluster of patterns with consistently high crime (high mean, low variance), while a low M_c value could indicate highly volatile or low-intensity patterns. This ranking is used to present the Top 5 and Bottom 5 most significant clusters in the visual analysis tool.

4.6 Visualization Framework

To facilitate the exploration of these identified patterns, we developed an interactive visualization tool. The interface consists of four linked components: (a) a main timeline view visualizing the 144 monthly time points; (b) a dropdown menu allowing users to select one of the k clusters, which highlights the corresponding subgraphs on the timeline and map; (c) a cluster visualization panel displaying all subgraph geometries belonging to the selected cluster; and (d) a crime intensity grid heatmap.

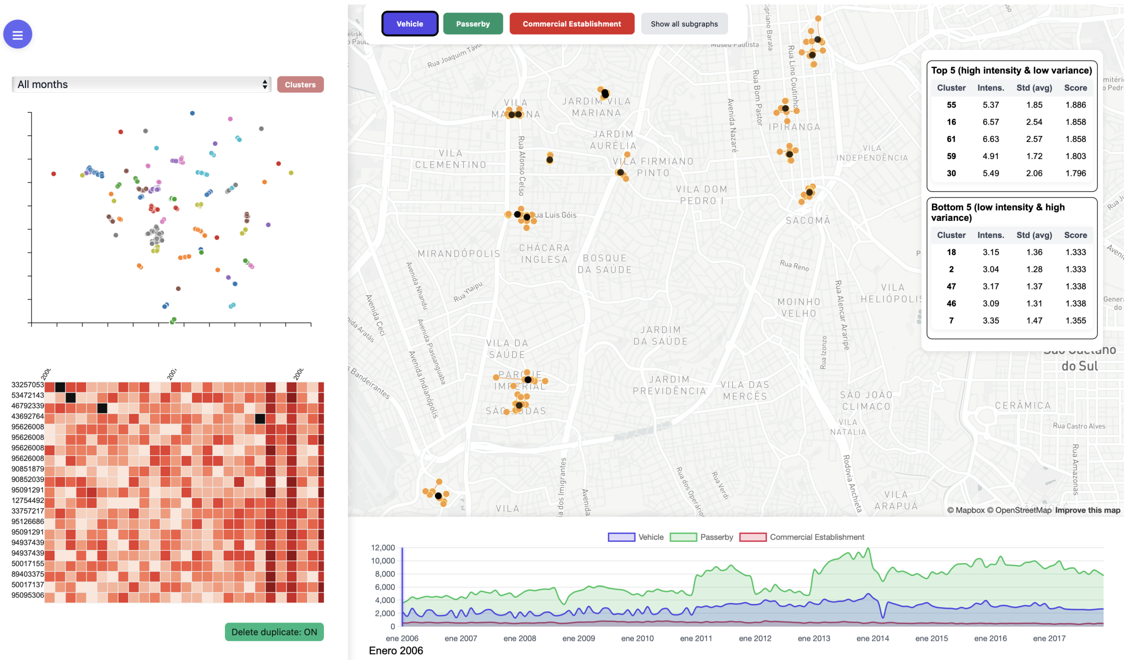

Figura 4.1: The interactive visualization tool displaying the analysis for a selected cluster. (A) UMAP embedding of all 1,440 subgraphs. (B) Temporal heatmap showing the activity of subgraphs within the selected cluster. (C) Geographic map plotting the location of the selected cluster's subgraphs. (D) Overall crime timeline. (E) Cluster ranking tables.

CHAPTER V

EXPERIMENTAL RESULTS AND ANALYSIS

This chapter presents the results from applying the methodological pipeline (detailed in Chapter IV) to the São Paulo crime dataset (2006-2017). The process successfully extracted, clustered and ranked 1,440 spatio-temporal subgraphs, revealing persistent and transient crime patterns.

5.1 Experimental Setup

The methodology was executed using the following parameters:

- **Data:** Crime reports from São Paulo (SSP) from January 2006 to December 2017.
- **Time Windows:** 144 monthly windows.
- **TDA:** $k = 10$ most significant "minima"(split tree) events were extracted per month, resulting in $1,440$ total subgraphs.
- **Clustering:** UMAP was used for dimensionality reduction (`n_components=2`), followed by DBSCAN (`eps=1.2`, `min_samples=10`) for clustering the 4D feature vectors (nodes, edges, lat, lon).

5.2 Cluster Identification and Ranking

The DBSCAN algorithm identified `61` distinct clusters from the 1,440 subgraphs. These clusters represent recurring spatio-temporal crime patterns.

To prioritize these patterns, we applied our ranking metric M_c (Equation IV.1) to identify clusters with high-intensity/low-variance (stable hotspots) and low-intensity/high-variance (volatile or cold spots). The results are summarized in Table 5.1.

CUADRO 5.1: Top 5 and Bottom 5 ranked crime clusters

Top 5 (High Intensity & Low Variance)			
Cluster	**Intens. (avg)**	**Std (avg)**	**Score (M_c)**
55	5.37	1.85	1.886
16	6.57	2.54	1.858
61	6.63	2.57	1.858
59	4.91	1.72	1.803
30	5.49	2.06	1.796
Bottom 5 (Low Intensity & High Variance)			
Cluster	**Intens. (avg)**	**Std (avg)**	**Score (M_c)**
18	3.15	1.36	1.333
2	3.04	1.28	1.333
47	3.17	1.37	1.338
46	3.09	1.31	1.338
7	3.35	1.47	1.355

5.3 Case Study: Analysis of High-Persistence Clusters

The Top 5 clusters represent the most stable and intense crime patterns identified. We can analyze them using the interactive visualization tool, as shown in Figure 4.1.

Analysis of Cluster 55 (Score: 1.886):

- **Geographic Analysis:** When selected, the tool plots the subgraphs for Cluster 55 on the map. These subgraphs are geographically concentrated in two neighborhoods. This indicates a chronic, geographically-fixed hotspot.

- **Temporal Analysis:** The timeline at the bottom shows the overall crime trends. However, the heatmap on the left provides a more detailed "fingerprint"for this specific cluster.

- **Significance:** This finding directly addresses a limitation of traditional methods. Instead of just a blob on a heatmap, we have identified a specific, recurring set of street segments (a subgraph) that consistently experiences high crime.

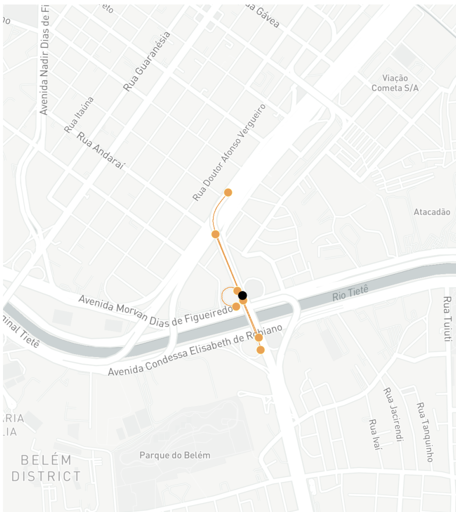

FIGURA 5.1: Subgraph 1 of Cluster 55

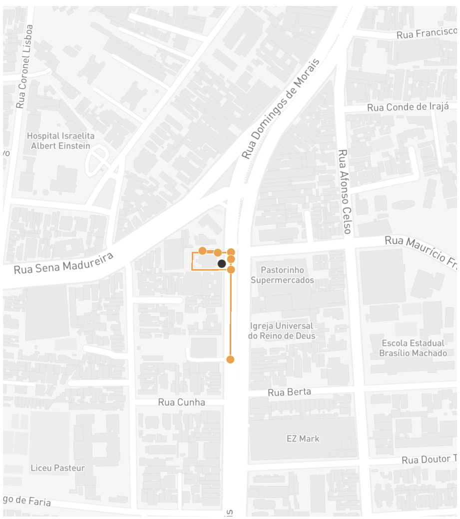

FIGURA 5.2: Subgraph 2 of Cluster 55

5.4 Temporal Persistence Analysis

A key finding of this experiment is the analysis of temporal persistence, as described by the user.

The temporal heatmap (Figure 4.1-B) visualizes the intensity of each subgraph *within the selected cluster* over the 144-month timeline. A crucial feature of this component is the ability to track **subgraph persistence**.

When we analyze the heatmap "with duplicates,"we can see instances where the *exact same subgraph ID* (e.g., '16588068') appears in consecutive months. This is a powerful finding:

1. **It confirms stability:** This is not just a similar pattern; it is the *identical topological structure* being identified as a significant crime event month after month.
2. **It quantifies çhronic":** We can now quantify this persistence. For example, "Subgraph '16588068' in Cluster 55 persisted for various consecutive months in 2011 and 2012."
3. **It validates the metric:** The 'Top 5' clusters, which our metric M_c ranked highly, exhibit this behavior most strongly. Their low standard deviation is a direct result of this consistent, repeating pattern. This aligns with the project's goal of quantifying the persistence of crime clusters.

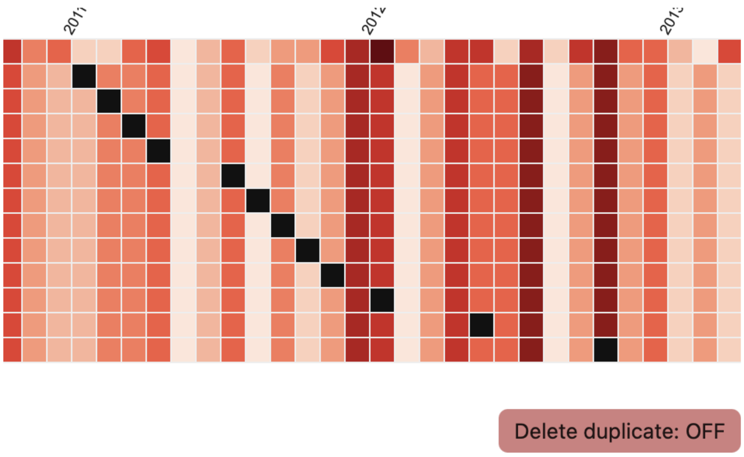

FIGURA 5.3: Delete duplicate off

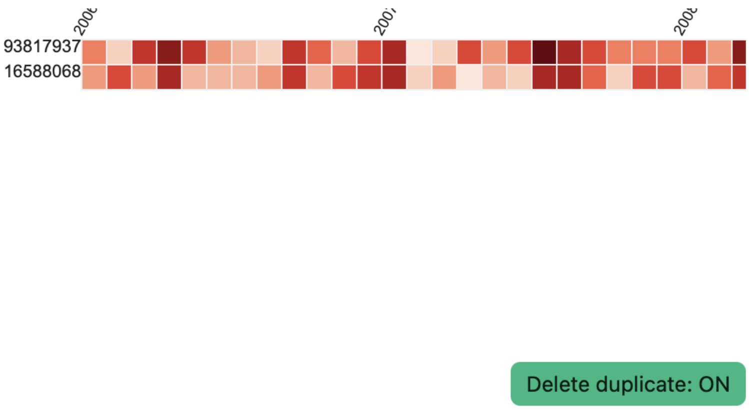

FIGURA 5.4: Delete duplicate on

5.5 Analysis of Volatile vs. Stable Patterns

By comparing the Top 5 and Bottom 5 clusters (Table 5.1), we can characterize different types of crime dynamics:

- **Top 5 (e.g., Cluster 55):** These are **chronic, stable hotspots**. They are geographically focused and temporally persistent. These are ideal targets for long-term, strategic police interventions and urban design changes.

- **Bottom 5 (e.g., Cluster 18):** These are **volatile or low-intensity patterns**. A high standard deviation (relative to the mean) suggests these are "flare-ups", like patterns that emerge and disappear quickly. They may be more geographically dispersed or represent transient, opportunistic crime, requiring different, more agile policing tactics.

This experiment validates the proposed framework by not only identifying clusters but, more importantly, providing a quantitative and visual method to differentiate them based on their temporal stability.

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